In mathematical language (S.G.)

Luca Pacioli stated that without mathematics there was no art. On the other hand his contemporary Erasmus of Rotterdam, in The Praise of Folly, reproached “philosophers” in these words: «They hold the unlettered public in particular disdain, when they confuse the ignorant with triangles, squares, circles and specially made geometric figures, placed one on top of the other to form a kind of labyrinth». In any case, polyhedra appear on more than one occasion in Venice between the fifteenth and the sixteenth century: think of the first small stellated dodecahedron of Saint Mark’s Basilica, attributed to Paolo Uccello and therefore preceding the work of Pacioli, or the stellated dodecicosahedron, the cuboctahedron, the great stellated dodecahedron and the two icosahedra, one regular and one stellated, in the inlaid wood of the Church of S. Maria in Organo, in Verona, without doubt inspired by Leonardo’s drawings. And again, perhaps also linked to Venice and to Pacioli, the mysterious polyhedron which appears in Dürer’s Melencolia I engraving, dated 1514.

After four centuries of silence, semiregular polyhedra reappear in the lithographs of Erasmo da Rotterdam, in the Elíogio della Follia, attributed to Leoardo da Vinci that Erasmus is referring to as “special geometric figures, placed one on top of the other to form a kind of labyrinth”. And again, perhaps linked to Venice and to Pacioli, the mysterious polyhedron which appears in Dürer’s Melencolia I engraving, dated 1514.

Maurits Cornelis Escher, più sporadicamente e limitatamente al piccolo dodecaedro stellato nelle sculture di Mimmo Paladino (Zenith, Carro, ecc.) e costituiranno la fonte di ispirazione di alcune tra le più note opere pittoriche di Lucio Saffaro. In realtà, verso la metà del secolo scorso, ci fu un isolato, esplicita incursione nella quarta dimensione con la celebre Crucifixion di Salvador Dalí (fig. 1). Se “si apre” (sarebbe più corretto dire “si sviluppa”) il cubo nel piano, nel modo abituale, si ottiene una figura costituita da sei quadrati (le sei facce del cubo) che simula una croce. Analogamente, se un ipercubo 4-dimensionale viene sviluppato nell’ordine spazio 3-dimensionale, si ottiene una figura composta da 8 cubi 3-dimensionali (fig. 2) che corrisponde alla croce utilizzata da Dalí nella sua opera.

Per rimanere nell’ambito delle 4 dimensioni non si possono menzionare le sculture geometriche di George W. Hart (fig. 3), professore presso il Dipartimento di Computer Science della Stony Brook University e, usando le sue stesse parole, freelance mathematical sculptor/designer10, ma con la stessa tecnica è possibile costruire altre affascinanti proiezioni, quali quella dell’iscocesso troncato realizzato alla Stony Brook University il 24 aprile dello stesso anno11. A questo punto è doveroso rendere omaggio a Alicia Boole Stott il cui primo lavoro, pubblicato nell’anno 1900, gettò le basi per la visualizzazione, attraverso opportune proiezioni, di politopi regolari 4-dimensionali, ma è anche interessante ricordare che i politopi regolari, cioè i parenti pluridimensionali degli ordinari solidi platonici, che come sappiamo sono 5, sono stati completamente classificati dai matematici: essi sono 6 in 4 dimensioni, ma diven- tano solo 3 dalla quinta dimensione in avanti. Anche queste strutture potrebbero riservare affascinanti sorprese per le arti visive, ma a mia


suitable projections, but it is also interesting to remember that regular polytopes, that are the multidimensional parents of the ordinary platonic solids, of which we know there are five, were fully classified by mathematicians: they are 6, four in dimensions, but they become only three from the fifth dimension onwards. These structures too hold fascinating surprises for the visual arts, but, to my knowledge, perhaps only Gian Marco Todesco has explored them. Going back to Hart, the ambigram Scott Kid created to write his name arouses curiosity and will continue to be a source for reflection: turn it 180° and one gets exactly the same image of the name “George Hart” (fig. 5). But not only regular polyhedra have attracted the attention of artists. Without wanting to cite generative and fractal art, where too often the algorithm has a far more predominant role than the artist, some of Escher’s works, among the most famous, are inspired by – or, it would be better to say, constitute extremely mathematically precise interpretations of – the Poincaré disk model of hyperbolic geometry, and there is no doubt that the Môbius strip, besides being directly depicted in some of his works, gave the Dutch artist suggestions for creating the worrying impossible structures.

Still, as we can see, we have always stayed in the field of geometry which is the most captivating aspect of the building of mathematics. Not by chance did Apollinaire claim that geometry is for the plastic arts what grammar is for the art of writing. But a not negligible proportion of the grammar of the “art” of mathematics is made up of those difficult formulæ which Erasmus’ talk of “letters arranged almost in battle order and variably manoeuvred” evokes. These “letters”, these symbols, arouse lively feelings of revulsion in some people, have they inspired or can they be associated with some form of visual art?

The first mathematical “letters”, the most familiar ones, are numbers and here we go back to two characters we met at the start of our journey: Albrecht Dürer and perhaps Luca Pacioli. In the work by Dürer we have already cited, Melencolia I, not only does an enigmatic solid appear, but the wall behind the winged figure is dominated by a magic square (fig. 6) which, like all magic squares, contains numbers arranged in a very precise order. Legend has it that it was Luca Pacioli himself who introduced the German artist to magic squares and that he had been profoundly irritated by Dürer’s appropriation of what he considered his creation.

In times much closer to our own, it was an Italian proponent of Arte Povera, Mario Merz, who remained fascinated by numbers, for more precise, the first numbers which constitute the Fibonacci sequence, and a leading role in some famous installations, like in the Guggenheim Museum in New York in 1971 or on the Mole Antonelliana in Turin in 1984. But the artist who more than any other demonstrated with his art that “that kind of labyrinth” and “those variably manoeuvred letters” from Erasmus’ inventive, far from “confusing the ignorant” can have effects of great aesthetic value is certainly Tobia Ravà. In his work numbers and symbols follow on from each other in an apparent absence of order, while

conoscenza, forse solo Gian Marco Tosdecso si è avventurato in queste inesplorate regioni. Ritornando ad Hart incuriosisce, e sarà legato ad alcune successive riflessioni, l’ambigramma che Scott Kim realizzato per scrivere il nome di George Hart: se lo si ruota di 180° si ottiene esattamente la stessa immagine (fig. 5).

Non solo i poliedri regolari hanno attratto l’attenzione degli artisti. Senza voler citare l’arte generativa e la frattale in cui troppo spesso il ruolo dell’algoritmo è di gran lunga predominante rispetto a quello dell’artista, le incisioni forse più note di Escher si ispirano, ma sarebbero più correttamente affermare che costituiscono delle interpretazioni artistiche, ma estremamente precise dal punto di vista matematico, del disco di Poincaré della geometria iperbolica ed è indubbio che il nastro di Môbius, oltre ad essere diretto protagonista di alcune sue opere, ha suggerito all’artista olandese la realizzazione delle inquietanti strutture impossibili.

Tuttavia, come si vede, siamo rimasti sempre nel campo della geometria che costituisce l’aspetto più accattivante dell’edificio matematico. Non a caso Apollinaire sosteneva che la geometria è per le arti plastiche quello che è la grammatica per l’arte dello scrittore.

Ma la grammatica per “l’arte” del matematico è costituita in misura non indifferente da quelle formule irte proprio di “lettere collocate quasi in ordine di battaglia e variamente manovrate” di cui parlava Erasmo. Queste “lettere”, questi simboli, che suscitano in molti sentimenti di vita repulsione, hanno ispirato o possono essere associate a qualche forma d’arte visiva?

Le prime “lettere” matematiche, quelle più familiari, sono i numeri e qui torniamo ad un paio di personaggi incontrati all’inizio di queste note: Albrecht Dürer e, forse, Luca Pacioli. Nella già citata opera del Dürer, Melencolia I, non appare solo un enigmatico solido, ma nella parete che si trova alle spalle della figura alata campeggia un quadro magico (fig. 6) che, come tutti i quadri matematici, contiene dei numeri disposti in un ordine ben preciso. La leggenda vuole che fosse stato proprio Luca Pacioli a far conoscere all’artista tedesco i quadri magici e che l’appropriazione da parte del Dürer di quello che il Pacioli riteneva sua creatura, lo avesse profondamente irritato⁶. In anni assai più vicini a noi fu un esponente italiano della corrente Arte Povera, Mario Merz, a rimanere affascinato dai numeri, per

Figure 5

Figure 6

Figure 7
Tobia Ravà, Soglia celeste. Courtesy Tobia Ravà.


still creating ineffably suggestive images (fig. 7): if we pay attention to a small portion of one of his paintings we lose ourselves in a labyrinth which we cannot grasp ... what it means. One could say that Ravà created the dream of mathematics in his works, where in the succession of formulae a complex, but rigorously logical, construction is materialized, which we can appreciate every fragment of which: a wonderful sensation which the mathematician clearly perceives, but struggles to make a non-mathematician feel part of.

What is more Ravà is also a mathematician (a conjecture is named after him), as too was Lucio Saffaro, and George Hart is a fully rounded mathematician.

One could object that Dürer’s numbers were nothing more than one of the elements used to represent the difficulties of alchemic transmutation; that Merz’s, closely linked to the logarithmic symbol, symbolized the growth energy of psychic matter; and that Ravà’s symbols belong not to mathematical but to Jewish iconography and together with the numbers refer to the kabbalah and the gematria.

Another objection that can be brought forward is that numbers are so universally exploitable as to be closer to the alphabet than to the apparent obscure signs which inhabit the nightmares of many students (and not just students).

All this is certainly true but the question which comes spontaneously is whether even mathematical symbolism can have some artistic value and, in order to answer that, we need to take one or two steps backwards.

When an artist creates a work what he certainly wants to avoid is indifference: he too wants to pass on emotions and ideas, he too wants to communicate, but it is essential that the emotions and ideas arounded in the audience correspond to those he wanted to pass on. There is therefore a utilitarian intention, which should not be present in the true work of art.

Should we blame then that there is no art in this creation? Not always, but sometimes, sometimes a logo can appear truly beautiful.

I would have preferred not to give examples, but I will make an exception: the famous Nike swoosh logo. As everyone knows, it is a stylized wing which, if on the one hand it reminds us of the Greek goddess of victory, on the other it passes on impressions of movement, agility and speed which are perfectly appropriate for a sports company.

But it is not so much its adequacy for the goal which prefixes what will indelibly strike people’s imagination. It is its essentiality that characterizes it. Succeeding in concentrating a myriad of meanings in a sign that is little more than a line makes such a logo a small work of art.

Yet that essentiality is the mathematical symbol’s fundamental characteristic: enclosing whole, sometimes very complex, concepts in a simple but suggestive symbol.

And here we come back to Erasmus and to an expression we had overlooked up until now: “to confuse the ignorant”.

I have maintained for many years that the aversion which a substantial part of the human race feels towards mathematical symbolism comes precisely from an “ignorance” of history. An often forgotten history, rich in copps de théâtre, in quick successes and equally quick declines: a history always dominated, and perhaps this will amaze people, by the demand for synthetic but exhaustive “communication”.

On the history of mathematical symbols, the work of the Swiss mathematician Florian Cajori is essential reading, even though, having been published in 1894, it omits all the modern symbolism which arose out of predicate logic and in particular from the work of Giuseppe Peano and the Bourbaki school.

ces to some which, in my view, stand out for their essentialism. The first two are the very famous symbols for addition and infinity: + and ∞. Why does one indicate addition with a cross and infinity with a flattened 8? We can all agree on their elegant appearance, but what might they suggest to us? We will start with addition: in Latin this function is indicated by adding the conjunction et between the addends (exactly like “and” now in Anglophone countries); amanuenses developed the habit of contracting the letter e to the point of making it disappear and at this point the move from t to cross seems quite natural. So + is no more than a kind of “shorthand contraction” of the Latin et. But what is most surprising is that the first + makes its appearance only in 1481 and the symbol will have to wait until the beginning of the eighteenth century. Recorde justifies his choice for the “equal” sign with exemplary synthetic clarity: “I will sette as I doe often in woorke use, a pair of parallels, or Gemowe lines of one lenghte, thus: =, because noe 2 thynges can be more equalle”.

As a second significant example, we can consider the triad of “greater”, “equal” and “lesser”; >, =, <. They have different dates: 1557 for the “equal” sign invented by Robert Recorde, 1651 for the “greater” and “lesser” ones created by Thomas Harriot. And they will have a troubled infancy, particularly the last two, since they will not be able to establish themselves decisively until the end of the eighteenth century. Recorde justifies his choice for the “equal” sign with exemplary synthetic clarity: “I will sette as I doe often in woorke use, a pair of parallels, or Gemowe lines of one lenghte; thus: >, because noe 2 thynges can be more equalles”. A success that is even more surprisingly late when one considers their plainness, that is, a smallest size in comparison with other historical symbols. A beautiful image created by a twelfth century primary school girl shown (fig. 8).

This essay’s quick tracking shot of history must include a homage to one of the greatest creators of symbols, Giuseppe Peano, an inspired mathematician from Turin who lived at the end of the nineteenth and start of the twentieth centuries. Predicate knowledge arose from profound criticism of the bases of the mathematical system, which appeared under- mined by the discovery of non-Euclidean geometries and of strongly pathological curves. We have observed at the start that mathematicians and artists like symmetries: polyhedrons and polytopes are an example. But they often express this preference with a touch of irony, as in the ambiguous dedicated to George Hart. They also like to play with symmetries and often they have a certain success, as the most common symbols of predicate logic demonstrate (existential and universal quantifiers, most likely only the first is Peano’s, fig. 9). In this case the suggestiveness derives from the letters of the alphabet which constitute the ini- tials of the concept one is seeking to express: an “upside-down” E for “it exists”, an A “rotated 180°” for “it exists”.

Figure 8: The symbols >, =, <.

Figure 9: Quantifiers.

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Figure 8: The symbols >, =, <.

Figure 9: Quantifiers.
symbol rather than the others to have success: in general symbols arise from the work of a single author, and then spread for various reasons, not least the technical exigencies of reproduction, which, if not properly understood, can cause interpretative difficulties and even misinterpretations.

In graphics (M.L.E.)

If, as we have stated so far, artistic value present at the same time, correlated and organically related to each other, so as to be indispensable to the communication we want to underline, different and distinct systems of grammatic and syntax have been used. At the same time, even graphic signs must be accessible and immediate, referring to shared and stabilised stereotypes, in the first case; in the second case they must be linked to a code, which the recipient necessarily has to know — if not, the message will be useless, and perhaps even damaging if it is misunderstood and misinterpreted.

Finally, the factor of perception, which can be presented at the same time, correlated and organically related to each other, so as to be indispensable to the construction we want to underline, different and distinct systems of grammatical and syntactic have been used. At the same time, even graphic signs must be accessible and immediate, referring to shared and stabilised stereotypes, in the first case; in the second case they must be linked to a code, which the recipient necessarily has to know — if not, the message will be useless, and perhaps even damaging if it is misunderstood and misinterpreted.

12. Definizione di “arte” dalla Enciclopedia Treccani online: «In senso lato, ogni capacità di agire e di produrre, basata su un particolare complesso di regole e di esperienze conoscitive e teoriche. Nell’ambito delle regole e dei procedimenti svolgere un’attività umana in vista di determinati risultati. […] Nell’ambito delle cosiddette teorie del “bello”, o dell’estetica, si tende a dare al termine “arte” un significato privilegiato, per indicare un particolare prodotto culturale che comunemente si classifica sotto il nome delle singole discipline di produzione, pittura, scultura, architettura, così come musica o poesia. Secondo il Garzanti online (http://www.garzantilinguistica.it/ricerca?q=arte): attività umana volta a creare opere a cui si riconosce un valore estetico, per mezzo di forme, colori, parole o suoni: l’arte della scultura, della pittura, così come musica o poesia».


14. È sufficiente qui ricordare il caso dei segnali di sicurezza, in particolare quelli stradali che presentano contaminazioni di genere (immagini, segni, eccetera) e conseguentemente presentano usi diversi delle tipologie di linguaggio, da quello tecnico a quello figurativo abbinati per semplificazione.

lead to a more or less empathetic response, all other things (satisfaction of the rules, use of the same techniques and transmission of analogous messages, or which can show what we have defined as “artistic value”) being equal.

There are many famous examples which are shared knowledge and which can be referred back to, in one context or another. Textual visual communications: logotypes, illustrated alphabets, dedicated language (e.g. Braile). Semiotic visual communications: brands, pictograms, dedicated language (e.g. for the deaf). Each of the categories we have listed is structured in its turn and can include iconic or symbolic, indicative/prescriptive or decorative/compositional signs. The greater or lesser possibility of artistic value will depend on the response of the recipient, on his potential emotional response and “instinctive” adhesion, which may depend on factors completely external to the graphic composition itself. One thinks for example of the sign-brand of the Red Cross, almost universally recognised and with a strong empathetic charge which depends not at all on the structure of the sign and entirely on the image it has the power to evoke: pain and danger, and at the same time help, support and solidarity. Or, again, of how significantly the emotional charge increases when the sign is on top of an ambiguous image (or vice versa), which we have seen in talking about mathematical and which also occurs where graphic production is concerned: this phenomenon occurs when an image apparently disconnected from meaning nevertheless comes to acquire a shared significance due to a concrete suggestion, which is therefore a “real” suggestion in the sense of collective heritage, and can be visualised in a way which is shared. Our example for mathematics was of the “greater” and “lesser” signs; in graphics, an example could be the Nike brand, which makes visible the trajectory “drawn” in the air by a tennis player, while configuring a completely abstract sign which can be perceived as a symbol. What emerges clearly from these synthetic observations is the role that good graphic planning plays not only for the necessary success of the communication of the message, but also for the possibility of a qualitative transformation of the message, to the point where it represents a shared element of the “artistic” heritage, even understood in the broad sense of the definitions we have cited: one thinks of the history of advertising graphics (in itself potentially a long way from the world of art) and of the contribution which some authors have made to it, transforming the creative sign into an element which certainly responds to the brief of the commissioner (launching a new product, exalting some characteristics of it, promoting publicity events, publicising the commissioner’s identity and so on) but is also a piece of art. The most glaring example, in this sense, is that of posters, which characters in art history have applied themselves to, for example Depero.

Maria Linda Falcidieno, Saverio Giulini – Signs and symbols between graphics and mathematics


used all three design components, privileging now the text used as image, now image alone, now signs. But just as unforgettable and fully possessed of artistic value is the work left by Armando Testa who, for example with the Punt e Mes brand, has transformed an abstract sign in (geometric) code, the sphere, into a visual translation of the text and therefore of a sign in (geometric) code, the sphere, into a visual translation of the text used as image, now image alone, now the text used as image, now image alone.

We still need to mention the system of work safety signs created by Eugenio Carmi for the Italsider factory in Genoa. In this case, too, geometric forms are used to translate concepts and parts of the body: here, the additional graphic sign of the chromatic colour field, without shades, brings everything onto the two-dimensional plane and makes critical reading homogenous with that carried out for mathematical symbols, while pointing out the role of colour as a captivating and attractive element.

Figure 12
International Red Cross.
The flag was adopted at the Geneva Congress of 1863 and in honor of the host country, with the inverted colors, the Swiss flag was chosen. The purpose was to assist the military and civilians involved in wars or natural disasters.

Figure 13
Stills now the red cross symbol is related to a concept of help and solidarity, emphasized by the sound call.

Figure 14
The red cross, with certain proportions, but with some change or addition, becomes just an indication.

Figure 12
Croce Rossa Internazionale.
La bandiera fu adottata al Congresso di Ginevra del 1863 e in onore del paese ospitante fu scelta, con i colori invertiti, la bandiera svizzera. Lo scopo era l’assistenza ai militari e ai civili coinvolti in guerre o disastri naturali.

Figure 13
Ancora oggi il simbolo della croce Rossa è associato a solidarietà e aiuto, enfatizzato dalla sensazione acustica della sirena.

Figure 14
La croce rossa, con certe proporzioni, ma con modificazioni e aggiunte diviene un semplice segno indicativo.

In conclusion, signs of mathematics and signs in image follow – seppure forse inconsciamen- te – the medesime "regole" della comunicazio- ne visiva, sia per ciò che attiene la loro intro- duzione, sia per quanto riguarda la successiva affermazione; la diffusione e la permanenza del segno, in un caso come nell’altro, dipende- no direttamente dalla possibilità di percepire, recepire, tradurre e comprendere nel minor tempo possibile detto segno, senza sforzi men- monici, ma al contrario “facendolo proprio” e, quindi, non dimenticandone più il signi- ficato, né tentonsenlo la forma.

D’altra parte, come visto in precedenza, la co- municazione visiva utilizza a volte concetti (la specificità delle lettere, ad esempio, o la raf- figurazione della velocità come nell’esempio della Nike), a volte figure bi o tri-dimensionali (cerchio, sfera, ecc.), fino a giungere all’utilizzo di simboli (la croce); l’interpretazione del messaggio deve essere sempre e comunque univoca, in relazione al contesto e in ciò gioca un ruolo determinante il fattore sinestetico, come chiaramente evidenziato in alcuni esem- pi fatti in precedenza.

La lettura di alcuni segni grafici, insomma, si può alternare anche all’interno del medesimo messaggio visivo, con una possibile interpretazione di icona o simbolo, che dipende certamente dalle presenze e dalla tecnica di traduzione linguistica, ma anche dallo specifico trattamento che l’inventiva dell’autore dà come contributo personale e che trasforma il messaggio nella sua essenza stessa (figg. 12, 13, 14) creatività, dunque, la medesima che ha contraddistinto la definizione dei simboli matematici, spesso anche fuori dagli schemi più stereotipati.